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Comparative study of confined concrete models

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Abstract— Analytical models for the full stress–strain relationship of confined concrete in compression are required for the numerical simulation of the structural behavior of reinforced concrete structural elements. There are many analytical models presented in the literature, which are generally empirical and are based on tests on reinforced concrete columns. This paper reviews some widely used analytical models calibrated using test results on reinforced concrete and compares their predictions with assumed data on uniaxial compressed column. The predicted model for the peak stress, corresponding strain to peak stress and stress-strain relation are also compared. The present investigation focuses only on those models de-veloped for internal confined concrete by normal and high yield steel ties. All the well-documented analytical models proposed so far are applied to predict the results of assumed data.

Index Terms— Column, concrete, confinement, models, peak-stress and strain.

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1 Introduction

THE reinforced concrete columns are the main load-bearing elements in reinforced concrete (RC) structure, as the column has to with stand the entire load and transfer it to foundation.The studies on the behaviour of RC short columns subjected to axial and eccentric loads were started in the early 1900s byConsidère [3],Talbot[25] and Withey[26]. The fundamental concept of confinement was pioneered in 1927. Problems of confined concrete have long been recognized and investigated both experimentally and analytically in the past. Several confinement models have been developed to predict the stress-strain behaviour of normal, as well as high strength concrete. All the proposed models are based on previous experimental work where several parameters have been derived from the statistical processing of the results. Depending on how confining stresses are provided to concrete are categorized as being active or passive. Confinement applied by prestressing on a concrete core is of active type. On the other side, lateral reinforcements such as spirals or ties provide passive confinement activated by the expansion of concrete. In the University of Illinois under the guidance of Richart et al. [16] introduced the term lateral confining stress and proposed a model to predict the confined concrete strength. From this point onwards many linear and non-linear models were developed to represent the stress-strain behavior of confined concrete. The present investigation focuses only on those models developed for internal confined concrete by normal and high yield steel ties. All the well-documented analytical models proposed so far are applied to predict the results of assumed data.

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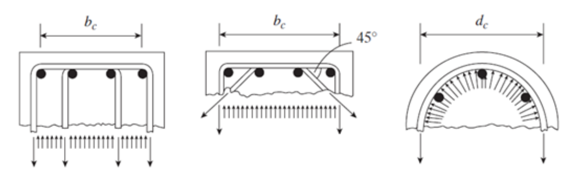
2 Confinement mechanism

Confinement effect in concrete depends upon two factors the tendency of concrete to expand and the lateral stiffness of the confining medium to resist the expansion of concrete. Ductility increases in concrete on confinement effect. The basis of this approach is that the additional ductility available in confined concrete is due to the energy stored in the confining medium. To obtain lateral confining pressure it must satisfy two conditions the strain compatibility between the concrete and the transverse ties and equilibrium of forces in the free-body diagram for any sector of the confined section. The second condition leads to the following relationship between the lateral confining stress (f_l) and the yield strength of transverse steel (f_{yt}).



$f_l = \frac{4 A_{st} f_{yt}}{s\, b_c}$ $\quad f_l = \frac{3.14 A_{st} f_{yt}}{s\, b_c}$ $\quad f_l = \frac{2 A_{st} f_{yt}}{s\, d_c}$

Fig. 1. Lateral confining pressure provided by different arrangements of transverse reinforcement (Source: N.Subramanian 2011).

3 Confinement effect in RC column

The effectiveness of confinement in RC column depends on reinforcement arrangements and concrete properties. The circular spiral confines concrete effectively because they provide continuous confining pressure around the circumference of concrete. But square hoops can apply only confining reaction near the corners of the load-bearing because the pressure of the concrete against the sides of the hoops tends to bend the side outwards. Therefore a considerable portion of the concrete cross-section may be unconfined.

Because of internal arching between the corners of the concrete is confined effectively only in the corners and central region of the section. Park and Paulay [13]. Concrete confinement is a three-dimensional phenomenon that cannot be reduced to a sectional level. Therefore, it is essential to consider the variation of lateral pressure along the member length. The distance between the longitudinal bar should be minimum and it should be placed tightly with the transverse steel or else it will reduce the effectiveness of confinement. Low-grade concrete is more ductile in nature hence they have better confinement.

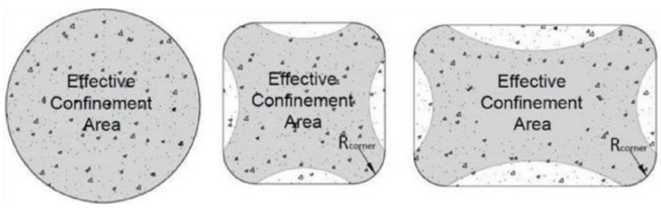


Fig. 2. Effective confinement area of circular, square and rectangular ties.(Source: ACI 440.2R-08)

4 Confinement Models

Many researcher developed linear and nonlinear strain-Strain model based upon their experimental results. Through we can clearly the Predict the behavior internally confined concrete such as peak confined concrete stress its corresponding strain and ductility of the confined concrete. The analytical models proposed by the following researchers are studied: Richart et al [16], Roy and Sozen [17], Kent and Park[10], Sheikh and Uzumeri[24], Scott et al.[22], Fafitis and Shah [6], Mander et al. [11], Yong et al. [27], Saatcioglu and Ravi [19], El-Dash & Ahmad[5], Cusson and Paultre [4], Mansur et al. [12], Hoshikuma et al.[9] and Assa et al. [1]. Notations used by the above researchers were reduced to the common notation for the sake of consistency and comparison. The stress-strain curves of the reviewed analytical models are illustrated in Figure 3, while the peak stress and corresponding strain of models listed in Table 1. Strain-strain relationship of modes are listed in Table 2 and confinement models are described subsequently according to their comparative importance first.

Richart et al. (1929) model was the first to capture lateral pressure greatly enhances the maximum strength of confined concrete. A linear relationship was suggested to find peak stress of passive confined concrete based on active confinement produced by oil pressure through a hand pump.

$$fcc = fuc + 4.1\ fl \qquad (1)$$

Roy and Sozen (1964) conclude that the confinement provided by rectilinear ties does not enhance the strength of the confined concrete and there was a considerable increase in ductility of the concrete. They proposed a stress-strain curve of two straight line that ascending branch meeting at peak concrete stress fc at corresponding strain of 0.002 and descending branch straight line meets 50% of peek concrete stress at defined strain point ε_{50} .

Kent and Park (1971) also assume that the strength of confined and unconfined concrete is the same and proposed stress-strain model for confined and unconfined concrete as suggested by Roy and Sozen [17]. They represented that the ascending branch of the stress-strain curve start from origin then increases in form of second degree parabola was not affected by confinement. The descending branch was a function of lateral steel, spacing of ties and core concrete area. They concluded that confinement effect of rectangular tie increases concrete strength is very small. The ascending and descending branch in the model curve are expressed by two different equations.

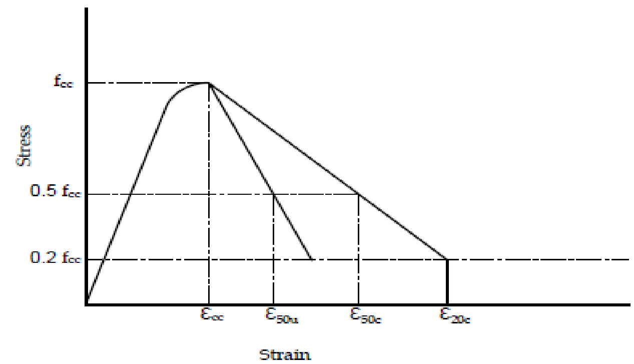


Fig. 3. Model proposed by Kent and Park

Sheikh and Uzumeri (1982) proposed a stress-strain model based on effectively confined concrete area which is less than the core concrete area enclosed by the center line of the perimeter tie. They considered effect of longitudinal bars and tie spacing in confinement model. A stress-strain relationship for confined concrete proposed was contain ascending branch upto ε_{s1}, is a second degree parabola, horizontal branch between ε_{s1} and ε_{s2} and the descending branch is suggested upto 30 percent of the maximum stress after which a horizontal line represents the concrete behavior.

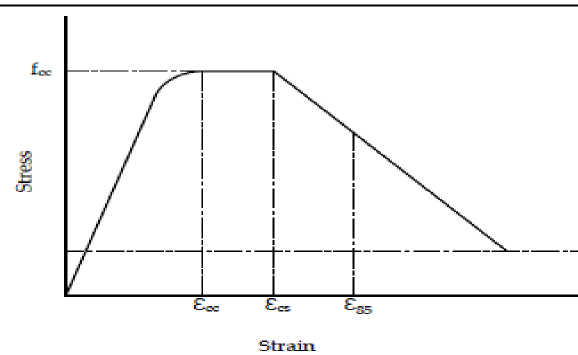


Fig. 4. Model proposed by Sheikh and Uzumeri

Mander et al. (1994) proposed a unified stress-strain model for confined concrete subjected to uniaxial compressive loading applicable to both circular and rectangular sections. The stress-strain curve is based on an equation suggested by Popovics [29]. Effective confining pressure and the confinement effectiveness coefficient was calculated similar to the one used by Sheikh and Uzumeri [24] where confining stress is fully developed due to arching action.

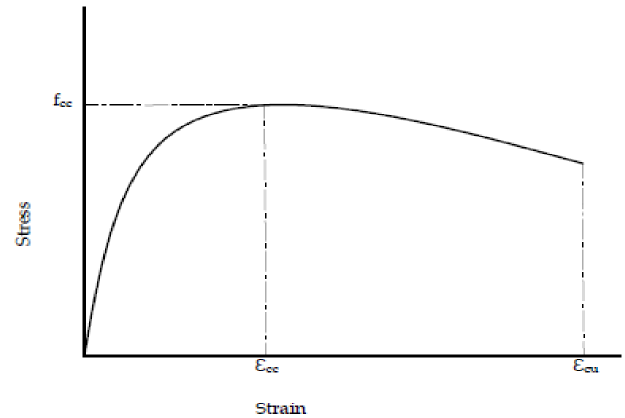


Fig. 5. Model proposed by Mander et al.

Yong et al. (1988) proposed an analytical model to construct a stress-strain relationship was based on experimental work done for high strength columns with rectangular ties. Stress-Strain behavior was studied with respect to the effects of the volumetric ratio of lateral ties, the concrete cover, and the distribution of the longitudinal steel around the core perimeter. The ascending and the descending branches in the model curve are expressed by two different polynomial equations.

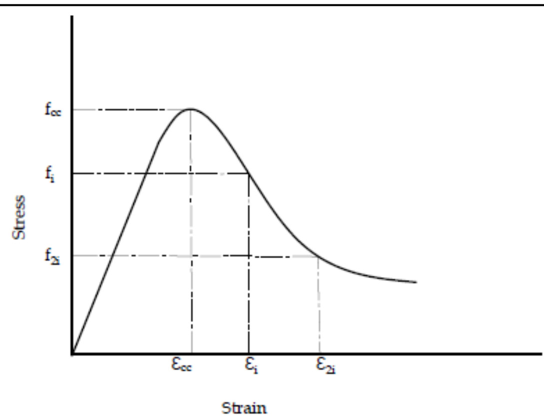


Fig. 6. Model proposed by Yong et al

Saatcioglu et al. (1992) proposed an analytical model to construct a stress-strain relationship for confined concrete based on equivalent uniform confinement pressure generated by the reinforcement cage. Combination of lateral pressure and axial compression results in a tri-axial state of stress. Transverse strains caused by lateral pressure counteract the tendency of material to expand laterally and result in increased strength. They represented a second order parabola for the ascending branch, a linear descending branch and a constant residual strength equal to 20% of peak strength.

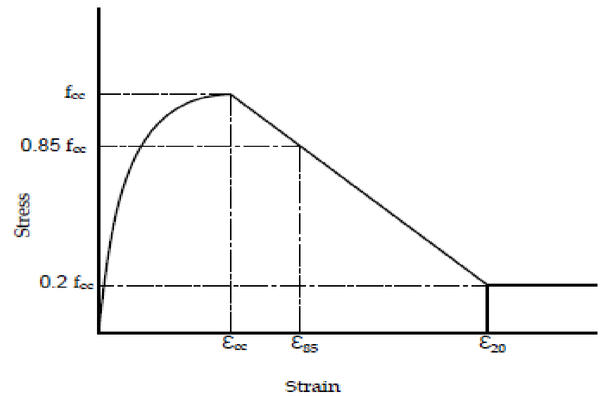


Fig. 7. Model proposed by Saatcioglu et al.

Cusson and Paultre (1995) built their model for confined high-strength concrete based on the actual stress in the stirrups upon failure and they did not consider the yield strength, as the experimental work have shown that the yield strength for the transverse steel is reached in case of well confined columns. The ascending and the descending branches in the model curve are expressed by two different equations.

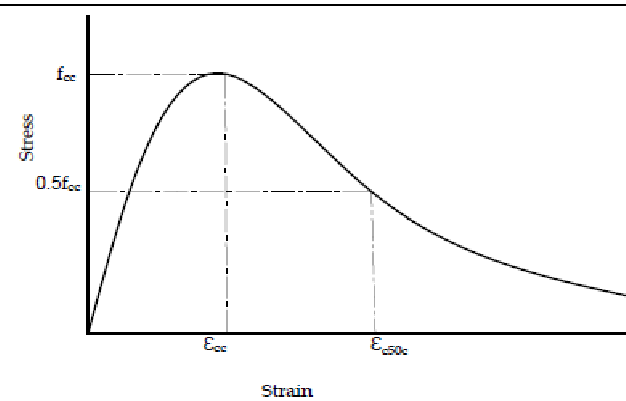


Fig. 6. Model proposed by Yong et al Cusson and Paultre

TABLE 1

PEAK STRESS AND CORRESPONDING STRAIN OF CONFINEMENT MOELS

Reserarcher	Peak stress (f_{cc})	Peak Strain (ε_{cc})
Sheikh and Uzumeri	$K_s 0.85\, f_{uc}$ $K_s = 1 + \frac{B'^2}{140\, P_{occ}} [\left(1 - \frac{nC'^2}{5.5B'^2}\right)(1 - \frac{s}{2B'})^2]\sqrt{p_s f_y}$	$80\, K_s\, f_{uc}\, 10^{-6}$
Fafitis and Shah	$f_{uc} + [1.15 + \left(\frac{3048}{f_{uc}}\right)] f_l$	$1.027\, 10^{-7} f_{uc} + 0.0296\left(\frac{f_l}{f_{cc}}\right) + 0.00195$
Mander et al.	$f_{uc}[2.254\sqrt{1 + 7.94(\frac{f_l}{f_{uc}})} - 2\left(\frac{f_l}{f_{uc}}\right) - 1.254$	$\varepsilon_{co}[\,1 + 5\,(\frac{f_{cc}}{f_{uc}} - 1)]$
Yong et al.	$[(1 + 0.0091(1 - \left(\frac{0.245\, s}{B}\right)\left(p_s + \frac{n\, d'_{st}}{8\, s\, d_s}\, p_l\right)\left(\frac{f_{yh}}{\sqrt{f_{uc}}}\right)] f_{uc}$	$\frac{0.00265 + 0.0035(1 - \frac{0.734\, s}{B})(p_s f_{yh})^{\frac{2}{3}}}{\sqrt{f_{uc}}}$
Saatcioglu et al.	$f_{uc} + 6.7(\,f_l\,)^{-0.17}\, f_l$	$\varepsilon_{co}[\,1 + 5\,K]$ $K = 6.7(\,f_l\,)^{-0.17}\,\frac{f_l}{f_{uc}}$
El-Dash & Ahmad	$f_{uc} + [5.1(\frac{f_{uc}}{f_{yh}})^{0.5}(\frac{d'_{st}}{p_s})^{0.25}]\, f_l$ $f_l = 0.5\, p_s f_{yh}(1 - \sqrt{\frac{s}{1.25\, d_s}})$	$\varepsilon_{co} + [\frac{66}{\left(\frac{s}{d'_{st}}\right) f_{uc}^{\;1.7}}]\,\frac{f_l}{f_{uc}}$
Cusson and Paultre	$f_{uc} + 2.1(\frac{f_l}{f_{uc}})^{0.7}$	$\varepsilon_{co} + 0.21(\frac{f_l}{f_{uc}})^{1.7}$
Mansure et al.	$f_{uc}[1 + 0.6(\frac{p_s f_y}{f_{uc}})^{1.23}]$	$\varepsilon_{co}[1 + 2.6(\frac{p_s f_y}{f_{uc}})^{0.8}]$
Hoshikuma et al	$f_{uc}[1 + 0.73(\frac{p_s f_y}{f_{uc}})]$	$0.00245 + 0.0122(\frac{p_s f_y}{f_{uc}})$
Assa et al.	$f_{uc}[1{+}3.36\frac{f_l}{f_{uc}}]$	$\varepsilon_{co}[1{+}21.5\,\frac{f_l}{f_{uc}}]$

TABLE 2

PEAK STRESS AND CORRESPONDING STRAIN OF CONFINEMENT MOELS

Reserarcher	Ascending Branch	Descending Branch
Kent and Park ®	$f_{uc}[\left(\frac{2\,\varepsilon_c}{\varepsilon_{co}}\right) - (\frac{\varepsilon_c}{\varepsilon_{co}})^2]$	$f_{uc}[1 - Z\,(\varepsilon_c - \varepsilon_{co})]$
Scott et al.	$kf_{uc}[\left(\frac{2\,\varepsilon_c}{0.002k}\right) - (\frac{\varepsilon_c}{0.002k})^2]$	$kf_{uc}[1 - Z_m\,(\varepsilon_c - 0.002k)]$
Fafitis and Shah	$f_{cc}[1 - (1 - \frac{\varepsilon_c}{\varepsilon_{cc}})^A]$	$f_{cc}\, exp[-k(\varepsilon_c - \varepsilon_{cc})^{1.15}]$
Mander et al.	$\frac{f_{cc}\, x\, r}{r - 1 + x^r}$	
Yong et al.	$Y = \frac{AX + BX^2}{1 + (A-2)X + (B+1)X^2}$	$Y = \frac{CX + DX^2}{1 + (C-2)X + (D+1)X^2}$
Saatcioglu et al.	$f_{cc}[\left(\frac{2\,\varepsilon_c}{\varepsilon_{cc}}\right) - (\frac{\varepsilon_c}{\varepsilon_{cc}})^2\,]^{\frac{1}{(1+2k)}}$	
Cusson and Paultre	$f_{cc}[(\frac{k\,\left(\frac{\varepsilon_c}{\varepsilon_{cc}}\right)}{k - 1 + (\frac{\varepsilon_c}{\varepsilon_{cc}})^k})$	$f_{cc}\exp[(K_1(\varepsilon_c - \varepsilon_{cc})^{K_2})]$
El-Dash and Ahmad	$Y = \frac{AX + (B-1)X^2}{1 + (A-2)X + BX^2}$	

4.1 Peak stress

Many researchers have indicated that the peak stress and peak strain are principally related to the effective confinement parameter ($fl\;/fuc$), yield strength tie, spacing of ties and confinement effective coefficient respectively, where f_l and fuc are the lateral confinement pressure and compressive strength of unconfined concrete. Confinement effective coefficient is different for every model determined by regression analysis of their experimental results. Richart et al.[16] based model and has been widely used to calculate the peak stress of confined concrete.

4.2 Peak strain

Peak strain is a major component of predicting descending and ascending branches, and accuracy of the stress-strain curve is highly sensitive to the peak strain. Therefore, predicting an accurate peak strain results in a highly accurate stress-strain model. Similar to the peak strength, it is well established in the literature that the peak strain of confined concrete has a relationship with lateral confining pressure ($fl\;/fuc$), and peak strain of unconfined concrete (ε_{co}).

4.3 Stress-Strain relationship

The main application of the analytical stress–strain relationship is in the analysis for the load-deformation response and ductility of reinforced concrete columns. The earliest analytical models are attributed to Hognestad [8] who used a parabolic expression for the stress strain relationship. Popovics[29] and Sargin et al. [21] used a mathematical fractional function for the stress strain relationship. El-Dash and Ahmad [5] model based on Sargin et al.[21]. Yong et al.[28] used two polynomial equation for the ascending and descending part of the stress-strain curve each similar to Sargin et al.[21]. Kent and Park [10], Scott et al.[22] and Sheikh and Uzumeri [24] used a fractional equation for the ascending part of stress strain curve and a linear function for the descending part of the curve. Mander et al [11] used a mathematical fractional function for the stress strain relationship based on Popovics[29] model .Saatcioglu and Razvi [19] used parabolic expression for the stress strain relationship identical to Hognestad [8]. Cusson and Paultre [4] used the mathematical fractional function originally proposed by Popovics [29] for ascending branch and mathematical exponential function similar to Fafitis and Shah[6] for descending branch.

4.4 Ascending branch

The ascending branch consists of peak stress (fcc), peak strain (εcc), and initial tangent modulus (Et) of concrete. Ascending branch of all confinement models indicates the same general trend behavior initially linear was followed by curvature up to the peak stress.

4.5 Descending branch

Ductility of confined concrete is determined by descending branch of stress-strain response. Descending branch is expressed using three points of the stress-strain behaviour of confined concrete. The first point is related to the peak strain, εcc, and peak stress, fcc, of confined concrete. The second point or intermediate point is related to strain corresponds to 85% or50% confined concrete strength this point. Third point is related to ultimate strain of confined concrete. The ultimate strain is considered as the concrete strain at first hoop fracture. Mander et al.[11] proposed expressions to predict ultimate strain based on energy balance methods. Researchers [10,24] considered strain corresponds to 20% and 30% confined concrete strength as ultimate strain.

5 Application of Confinement Model

A analytical work was carried out for calculating stress- strain curve of various model and to predict peak stress and strain of various models it required assumed input data and assumption. Input data required such uniaxial compressive strength of concrete fc and corresponding axial strain assumed to be 0.002, properties of longitudinal and lateral steel, cross-sectional dimensions of specimens, diameter of lateral bar and spacing of ties. To compute the lateral confining pressure stress in lateral steel is assumed equal to its yield strength.

The analysis procedure involves following steps

- Assume input data mentioned above
- Calculate lateral confining pressure f_l by using Mander et al. model
- Obtain calculated confined concrete stress for corresponding strain values.
- Vary the axial strain in concrete from ε_0 to ε_{cc} and ε_{cc} to ε_{cu}.
- Draw stress-strain curve and plot peak stress-strain point for linear confinement models by above procedure.

Assumed data as follow uniaxial compressive strength of concrete f_{uc}= 30 MPa and yield strength of lateral steel f_y = 716 MPa , square cross-section of 150mmx150mm, 4 numbers of 12 mm dia. bar, 4mm diameter bar at 50 mm spacing and clear cover as 20 mm. To study about the confinement behavior due variation of spacing of ties and unconfined concrete strength assumed properties for three different spacing (50mm,100mm,150mm) and concrete strength (20 MPa and 30 MPa) results were discussed in table 3.

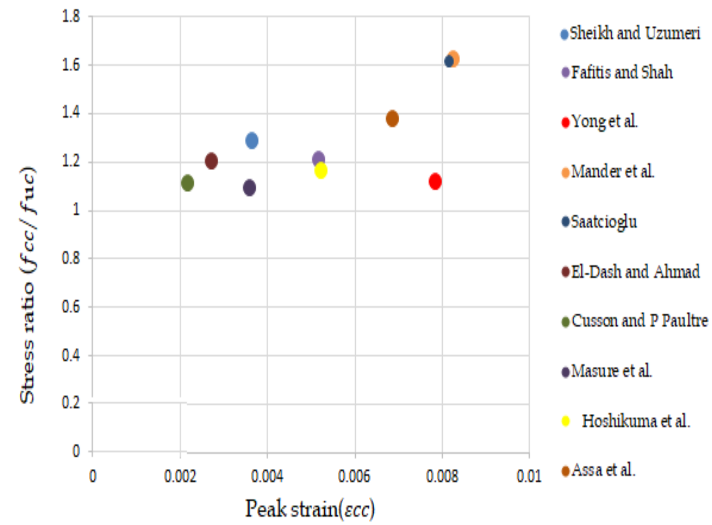


Fig. 7. Comparison of peak stress ratio at its corresponding strain

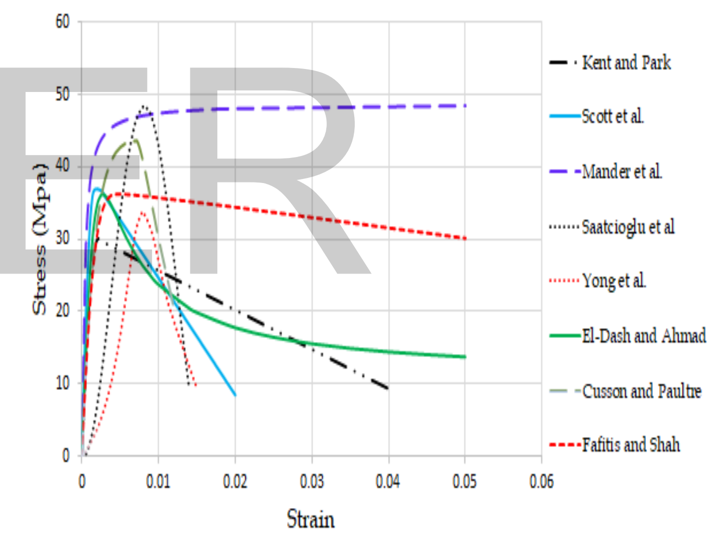


Fig. 7. Comparison of Stress-Strain curve

TABLE 3

PEAK STRESS AND CORRESPONDING STRAIN VARIATION OF CONFINEMENT MOELS FOR DIFFERENT PROPERTIES OF SPECIMEN

Properties	$f_{uc} = 30\ Mpa$ S= 50 mm		$f_{uc} = 30\ Mpa$ S= 100 mm		$f_{uc} = 30\ Mpa$ S= 150 mm		$f_{uc} = 20\ Mpa$ S= 50 mm	
Reserarcher	$\left(\frac{f_{cc}}{f_{uc}}\right)$	ε_{cc} $10^{-0.3}$ mm	$\left(\frac{f_{cc}}{f_{uc}}\right)$	ε_{cc} $10^{-0.3}$ mm	$\left(\frac{f_{cc}}{f_{uc}}\right)$	ε_{cc} $10^{-0.3}$ mm	$\left(\frac{f_{cc}}{f_{uc}}\right)$	ε_{cc} $10^{-0.3}$ mm
Sheikh and Uzumeri	1.28	3.68	1	2.82	0.9	2.5	1.5	2.83
Fafitis and Shah	1.21	5.17	1.1	3.91	1.07	3.44	1.37	5.9
Mander	1.63	8.25	1.35	5.46	1.24	4.40	1.86	10.6
Yong et al.	1.12	7.82	1.05	5.89	1.03	5.07	1.15	8.99
Saatcioglu et al.	1.61	8.15	1.34	5.46	1.24	4.47	1.92	11.23
El-Dash & Ahmad	1.21	2.71	1.04	2.06	1	2	1.25	4.12
Cusson and Paultre	1.08	2.08	1	2	1	2	1.11	2.16
Mansur et al.	1.09	3.58	1.04	2.91	1.02	2.66	1.15	4.19
Hoshikuma et al	1.17	5.21	1.08	3.83	1.05	3.37	1.24	6.59
Assa et al.	1.38	6.86	1.19	4.43	1.13	3.62	1.57	9.3

6 ANALYSIS OF RESULT

The analytical models was based on the computation of the four key parameters. These are : i) the maximum confined concrete strength, fcc ii) the confined concrete strain at the maximum strength, iii) the strain along the ascending branch of the confined stress-strain curve iv) the strain along the descending branch of the confined stress-strain curve v) the confined concrete strain when the stress drops to 30percent of the maximum confined concrete strength,ε0.50fcc while the strain at 0.3fcc is usually close to the point of failure due to hoop fracture and/or shear failure of the confined core.

The predicted confined concrete strength was found to be relatively higher for most models except Roy and Sozen[17], Kent and Park[10] models there is negligible enhancement in concrete capacity by confinement effect. The coefficient of variation confined concrete strength ranging from 8 to 63%. Unified models proposed by Mander et al. [11] and Saatcioglu et al. [19] have higher coefficient of variation ranging about 63 and 61%. The models proposed for high-strength concrete square column by Cusson and Paultre[4] have coefficient of variation about 8% then the models proposed for normal-strength concrete square column by Sheikh and Uzumeri [24]have coefficient of variation about 28%. This is due to fact ductility factor of concrete is considered in confinement models. Confinement models proposed for normal-strength concrete cannot used for high-strength concrete.

The predicted peak strain (ε_{cc}) corresponding to the peak strength (f_{cc}) of confined concrete models range from 2.71 to 8.25 x $10^{-0.3}\ mm$ whereas peak strain for unconfined concrete is less than or equal to 2 x $10^{-0.3}\ mm$. Models predicted higher peak strain value of concrete which is due to confinement effect it shows that there is enhancement in ductility.

Stress –strain relationship expression proposed by the researcher are well detecting stress-strain curves of their respective models. Ascending branch for all models are similar initially linear was followed by curvature up to the peak stress. Descending branch for models are different for various models. Researchers [10] proposed expression given linear decay for descending branch. Researchers [5, 19, 22 and 28] proposed expression given parabolic decay for descending branch. Mander et al.[11] stress-strain expression does not predicted any decay in the descending curve. Cusson and Paultre [4] predicted Steep decay in confined concrete strength in the descending curve which is not similar to proposed model.

Confinement effect is affected by varies parameters such among them spacing of ties and unconfined concrete strength are the crucial parameters. Results from table 3 show that by increasing spacing of ties and unconfined concrete strength confinement effect is reduced. Smaller spacing of ties leads to more effective confinement. The concrete is confined by the arching action between ties and if the spacing is larger the large volume of concrete is unconfined this factor is consider in all by reviewed confinement models. Ductility of low-strength concrete is more than high-strength concrete. Hence low-strength concrete has higher confinement effect than high-strength concrete.

7 CONCLUSION

The study of Several confinement models show that;

- Three key parameters affecting the shape and magnitude of stress-strain curve of concrete are the peak stress, the strain at peak stress, and the ultimate strain.
- It can be concluded that there are six key parameters primarily influence the effectiveness of lateral con-

finement. The most influencing parameter is found to be the spacing of ties and concrete strength.

- There is significant increase in the strength and ductility of concrete.
- Further study needs to be carried out in the future particularly in comparing experiment and analytical stress-strain behavior of confined concrete

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